Formula sheet EE-434 Power Electronics

Feel free to distribute it to your students. Please address any mistakes to hadeedahmedsher@gmail.com . Latex template by divisbyzero.com

Basic expressions

Average expression

$$X_{avg} = \frac{1}{T}\int_0^t x(t)dt \tag{1}$$

RMS expression

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} \tag{2}$$

$$X_{rms} = \sqrt{\sum_{n=1...}^{N} X_{n,rms}^2} \tag{3}$$

Fourier series

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)] \tag{4}$$

where,

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt \tag{5}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)cos(n\omega_0 t)dt \tag{6}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)sin(n\omega_0 t)dt \tag{7}$$

Power and Energy

$$Instantaneous Power p(t) = v(t)i(t) \tag{8}$$

$$Averagepower P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = \frac{W}{T} \tag{9}$$

$$Averagepower P_{dc} = V_{dc}I_{avg} \tag{10}$$

$$Apparent Power S = V_{rms}I_{rms} \tag{11}$$

$$S = \sqrt{3}V_{L-L,rms}I_{S,rms} \tag{12}$$

$$Reactivepower Q = V_{rms}I_{rms}sin(\theta - \phi) \tag{13}$$

$$Energy(W) = \int_{t_1}^{t_2} p(t)dt \tag{14}$$

$$W = \int_0^T p(t)dt \tag{15}$$

$$Powerfactor(pf) = \frac{P}{S} \tag{16}$$

Inductor

$$Voltage \implies v_L = L\frac{di}{dt} \tag{17}$$

$$Currenti_L = \frac{1}{L}\int v_L(dt) + I_{0-} \tag{18}$$

$$Energyw(t) = \frac{1}{2}Li^2(t) \tag{19}$$

$$\tau = \frac{L}{R} \tag{20}$$

Capacitor

$$Currenti_C = C\frac{dv}{dt} \tag{21}$$

$$Voltagev_c = \frac{1}{C}\int i_C(dt) + V_{0-} \tag{22}$$

$$Energyw(t) = \frac{1}{2}Cv^2(t) \tag{23}$$

$$\tau = \frac{1}{RC} \tag{24}$$

Phasors

$$Impedance(Z) = \sqrt{R^2 + (\omega L)^2} \tag{25}$$

$$\theta = tan^{-1}(\frac{\omega L}{R}) \tag{26}$$

Half wave rectifier with RL load

$$i_{(\omega t)} = \begin{cases} \frac{V_m}{Z}\left[sin(\omega t - \theta) + sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \beta \le \omega t \le 2\pi \end{cases} \tag{27}$$

For controlled rectifier

$$i_{(\omega t)} = \begin{cases} \frac{V_m}{Z}\left[sin(\omega t - \theta) - sin(\alpha - \theta)e^{(\alpha-\omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m sin(\omega t)d(\omega t) = \frac{V_m}{2\pi}(cos\alpha - cos\beta) \tag{29}$$

With free wheeling diode

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}sin(\omega_o t) - \sum_{n=2,4,6...}^{\infty}\frac{2V_m}{(n^2-1)\pi}cos(n\omega_o t) \tag{30}$$

With source inductance

$$V_o = \frac{V_m}{2\pi}(1 + cosu) = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right) \tag{31}$$

Full wave rectifier

Full wave rectified waveform fourier is

$$v_o(t) = \frac{2V_m}{\pi} + \sum_{n=2,4...}^{\infty}\left(\frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)\right)cos(n\omega_o t + \pi) \tag{32}$$

Ripple voltage for C filter

$$\Delta V_o \approx \frac{V_m\pi}{\omega RC} = \frac{V_m}{2fRC} \tag{33}$$

With LC filter
In CCM

$$V_o = \frac{2V_m}{\pi} \tag{34}$$

$$I_L = \frac{2V_m}{\pi R} \tag{35}$$

$$\frac{3\omega L}{R} > 1 \tag{36}$$

In DCM average inductor current.

$$i_L = \frac{1}{\pi}\int_{\alpha}^{\beta}\frac{1}{\omega L}\Big[V_m(cos\alpha - cos\omega t) - V_o(\omega t - \alpha)\Big]d(\omega t) \tag{37}$$

RLE load CCM

$$I_o = \frac{V_o - E}{R} = \frac{\frac{2V_m}{\pi} - E}{R} \tag{38}$$

$$v_o(t) = V_o + \sum_{n=2,4..}^{\infty} V_n cos(n\omega_0 t + \pi) \tag{39}$$

where, $VC_o = \frac{2V_m}{\pi}$ and $V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$

Full controlled with RL load

DCM

$$\begin{aligned} i_o(\omega t) = \frac{V_m}{Z}[sin(\omega t - \theta) - sin(\alpha - \theta) \\ e^{-(\omega t - \alpha)/\omega\tau}]for\alpha \le \omega t \le \beta \end{aligned} \tag{40}$$

$$\beta < \alpha + \pi \implies DCM \tag{41}$$

For CCM

$$\alpha \le tan^{-1}\left(\frac{\omega L}{R}\right) \tag{42}$$

$$v_o(\omega t) = V_0 + \sum_{n=1}^{\infty} V_n cos(n\omega_0 t + \theta_n) \tag{43}$$

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m sin(\omega t)d(\omega t) = \frac{2V_m}{\pi}cos\alpha \tag{44}$$

The harmonics are

$$V_n = \sqrt{a_n^2 + b_n^2} \tag{45}$$

where,

$$a_n = \frac{2V_m}{\pi}\left[\frac{cos(n+1)\alpha}{n+1} - \frac{cos(n-1)\alpha}{n-1}\right] \tag{46}$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{sin(n+1)\alpha}{n+1} - \frac{sin(n-1)\alpha}{n-1}\right] \tag{47}$$

$$V_o = \frac{2V_m}{\pi}cos\alpha \tag{48}$$

Three phase full wave rectifier

Uncontrolled with R load

$$v_o(t) = V_0 + \sum_{n=6,12,18,...}^{\infty} V_n cos(n\omega_o t + \pi) \tag{49}$$

where,

$$V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{m,L-L} sin(\omega t)d(\omega t) = \frac{3V_{m,L-L}}{\pi} \tag{50}$$

The amplitudes of the ac voltage terms are

$$V_n = \frac{6V_{m,L-L}}{\pi(n^2-1)} \tag{51}$$

for n=6,12,18,...

DC-DC converters - CCM

$$D = \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}}{T} = t_{on}f_s \tag{52}$$

$I_{max} = I_L + \frac{\Delta i_L}{2}$ and $I_{min} = I_L - \frac{\Delta i_L}{2}$.

Buck converter

$$V_o = DV_s \tag{53}$$

Minimum inductance L_{min}

$$L_{min} = \frac{(1-D)R}{2f} \tag{54}$$

$$L = \frac{V_o(1-D)}{\Delta i_L f_s} \tag{55}$$

Output capacitor C

$$C = \frac{1-D}{8L(\Delta V_o/V_o)f^2} \tag{56}$$

$\Delta i_L = \left(\frac{V_s - V_o}{L}\right)DT$

Boost converter

$$V_o = \frac{V_s}{1-D} \tag{57}$$

Minimum inductance L_{min}

$$L_{min} = \frac{D(1-D)^2R}{2f} \tag{58}$$

$$L = \frac{V_sD}{\Delta i_L f} \tag{59}$$

Output capacitor C

$$C = \frac{D}{R(\Delta V_o/V_o)f} \tag{60}$$

$\Delta i_L = \left(\frac{V_s}{L}\right)DT$

Buck-Boost converter

$$V_o = -V_s\left(\frac{D}{1-D}\right) \tag{61}$$

Minimum inductance L_{min}

$$L_{min} = \frac{(1-D)^2R}{2f} \tag{62}$$

$$L = \frac{V_sD}{f\Delta i_L} \tag{63}$$

Output capacitor C

$$C = \frac{D}{R(\Delta V_o/V_o)f} \tag{64}$$

$\Delta i_L = \frac{V_sDT}{L}$

Flyback converter

$$V_o = V_s\left(\frac{D}{1-D}\right)\left(\frac{N_1}{N_2}\right) \tag{65}$$

Minimum inductance L_{min}

$$(L_m)_{min} = \frac{(1-D)^2R}{2f}\left(\frac{N_2}{N_1}\right)^2 \tag{66}$$

$$L = \frac{V_sD}{f\Delta i_{Lm}} \tag{67}$$

Output capacitor C

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \tag{68}$$

$\Delta i_L m = \frac{VsDT}{L_m}$

Forward converter

$$V_o = V_s D\left(\frac{N_2}{N_1}\right) \tag{69}$$

Output capacitor C

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{RL_x Cf^2} \tag{70}$$

$\Delta i_L m = \frac{VsDT}{L_m}$

DC-AC inverters

Square wave inverter(RL series load)

$$i_o(t) = \begin{cases} \frac{V_{dc}}{R} + \left(I_{min} - \frac{V_{dc}}{R}\right)e^{-t/\tau} & \text{for } 0 \leq \ t \leq \frac{T}{2} \\ \frac{-V_{dc}}{R} + \left(I_{max} + \frac{V_{dc}}{R}\right)e^{-(t-T/2)/\tau} & \text{for } \frac{T}{2} \leq \ t \leq T \end{cases} \tag{71}$$

$$I_{max} = -I_{min} = \frac{V_{dc}}{R}\left(\frac{1-e^{-(T/2\tau)}}{1+e^{-(T/2\tau)}}\right) \tag{72}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)d(t)} = \sqrt{\frac{2}{T}\int_0^{T/2}\left[\frac{V_{dc}}{R} + \left(I_{min} - \frac{V_{dc}}{R}\right)e^{-t/\tau}\right]^2 dt} \tag{73}$$

$$v_o(t) = \sum_{1,3,5..}^{\infty} \frac{4V_{dc}}{n\pi} sinn\omega_0 t \tag{74}$$

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty}(V_{n,rms})^2}}{V_{1,rms}} = \frac{\sqrt{V_{rms}^2 - V_{1,rms}^2}}{V_{1,rms}} \tag{75}$$

Quasi Square wave inverter(RL series load)

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc}^2 d(\omega t)} = V_{dc}\sqrt{1-\frac{2\alpha}{\pi}} \tag{76}$$

$$v_n = \frac{2}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc} sin(n\omega_0 t)d(\omega_0 t) = \frac{4V_{dc}}{n\pi}cos(n\alpha) \tag{77}$$

$$V_1 = \left(\frac{4V_{dc}}{\pi}\right)cos\alpha \tag{78}$$

PWM inverter

$$m_f = \frac{f_{carrier}}{f_{reference}} = \frac{f_{tri}}{f_{sine}} \tag{79}$$

$$m_a = \frac{V_{m,reference}}{V_{m,carrier}} = \frac{V_{m,sine}}{V_{m,tri}} \tag{80}$$

Bipolar PWM inverter

$$v_o(t) = \sum_{n=1}^{\infty} V_n sin(n\omega_0)t \tag{81}$$

6 step inverter

$$V_{n,L-L} = \left|\frac{2V_{dc}}{3n\pi}\left[2 + cos\left(\frac{n\pi}{3}\right) - cos\left(\frac{n2\pi}{3}\right)\right]\right| for \quad n = 1, 5, 7, 11, 13... \tag{82}$$

3 phase PWM inverter

$$V_{n3} = \sqrt{A_{n3}^2 + B_{n3}^2} \tag{83}$$

where,

$$\begin{cases} A_{n3} = V_n sin\left(\frac{n\pi}{2}\right)sin\left(\frac{n\pi}{3}\right) \\ B_{n3} = V_n cos\left(\frac{n\pi}{2}\right)sin\left(\frac{n\pi}{3}\right) \end{cases} \tag{84}$$